AN ANALYSIS OF CURRENT NAVY PROCEDURES FOR FORECASTING DEMAND WITH AN INVESTIGATION OF POSSIBLE ALTERNATIVE TECHNIQUES

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THESIS

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ABSTRACT

An analysis is made of current Navy procedures for assigning a probability density function to demand and the technique(s) used to forecast the parameter(s) of the particular density function chosen. The gamma density function is investigated as a possible replacement for the three density functions currently being used. A comparison of the gamma and normal density functions is made with regards to inventory costs, observed protection levels, and unit effectiveness. Additionally, a comparison is made, through a simple simulation model, of demand patterns generated by demand reporting as it is done today and consumption data reporting as it might be done in the future. The intent of the simulation is to provide some insight into the impact the demand reporting method might have on the variance of demand.

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LIST OF SYMBOLS AND ABBREVIATIONS

A	Shape parameter of gamma distribution
AX	Fixed cost to place an order
α	Scale parameter of gamma distribution
C	Unit price of item
cdf	Cumulative distribution function
$\bar{D}$	Mean quarterly demand
FMSO	Fleet Material Support Office
H(r)	Risk for given r
I	Annual holding rate per unit of stock
ICP	Inventroy Control Point
τ	Average annual demand
MAD	Mean Absolute Deviation
$\hat{MAD}$	Estimate of MAD where $\hat{MAD} = 1.37\bar{D}^{.717}$
MAD_1	Estimate of MAD where $MAD_1 = \frac{\Sigma\lvert x-\bar{x}\rvert}{N}$
μ	Expected value of a random variable
Pr	Probability
π	Shortage cost
Q	Reorder quantity
r	Reorder level
S^2	Sample variance; $S^2 = \sum_{i=1}^{N} (x_i-\bar{x})^2/N-1$
S	Sample standard deviation; $S = \sqrt{S^2}$
SA	Estimate of S; SA = 1.25 MAD_1
SD	Estimate of S; SD = 1.25 $\hat{MAD}$

SMO	Exponentially smoothed mean demand
$\bar{x}$	Sample mean; $\bar{x} = \sum_{i=1}^{N} x_i/N$
THC	Total holding cost

I. INTRODUCTION

Today, the Naval Supply System faces the task of supporting very complex weapon systems under the burden of a funding climate that is already austere and will probably become more so. Accordingly, this situation makes efficient inventory management of utmost importance. This is not to say that inventory management has not been vital in the past, but rather the decision-making process with regards to inventory management becomes more critical. Obviously as the dollars the supply system receives are reduced, the application of the dollars must be judiciously applied to provide the best support possible to the fleet. By support it is meant the repair parts required to keep a weapon system or essential shipboard/aircraft operating. In view of the fact that an effective essentiality coding has not been established within the Navy, this means supporting almost everything with a Navy stock number. It should be noted that the general objective of the Naval Supply System Command for the past several years has been to reduce the time equipment is down or not operating due to the failure of a repair part. This of course entails more than having the part in inventory; however, this is the most critical aspect since most items required are unique to the Navy and are not readily available from industry.

Essential to having the right part on hand at the right time is the ability to predict future demand for repair parts. This is by no means an easy task in view of the random nature of demand over a given time interval. The purpose of this thesis is to appraise what is being done within the Naval Supply System to predict future demand and to investigate possible alternative techniques.

In predicting future demand there are two major factors that must be addressed. They are:

1. Determine the probability distribution that represents or at least approximates the actual distribution.

2. Develop a technique for forecasting the parameters of the probability distribution determined to be representative of how future demand is expected to be generated.

The current Navy methods of forecasting demand will be investigated. The selection of the probability distribution will then be discussed with primary emphasis on comparing the normal distributions presently in use with gamma distributions as to their effect on the establishment of inventory levels. A brief discussion is made with regards to the use of the Poisson, negative binomial and the pseudo-normal distributions. The method of demand collection is then examined, and some possible problems associated with the current system within the Navy are discussed. Finally, the conclusions and recommendations generated by this study are presented.

II. DEMAND FORECASTING

There are many ways to forecast demand; however, most of them can be grouped into two broad categories. The technical or subjective method and the scientific (some people would prefer mathematical) or the objective method. The technical method is the method by which the technical experts in a particular field give their best subjective estimate as to the demand for an item. This method appears appropriate only when there does not exist past demand data for an item. It could be useful possibly as a weighting factor in one of the scientific methods. Presently, the Navy supply system uses the technical method for computing Best Replacement Factors (BRF) for new items of supply or for items which have not realized any demand but are required to be stocked. This method will not be discussed any further, since the objective of this thesis is to deal with the items having recurring demand.

The scientific method of forecasting demand breaks down to basically two types of technique. The first includes the time series techniques where past demand patterns are used to predict future patterns. Some examples of this type are the straight average, moving average, single and double exponential smoothing, spectral analysis and Bayesian procedures. These techniques rely solely on looking at the pattern of past demand over time to predict

future demand for a specified time frame. The second type within the scientific method is the causal type. Although this type also relies on past demand patterns, it attempts to explain the causes for the particular patterns. Therefore, this technique requires the knowledge of certain causal variables which in turn can be related to the dependent variable for prediction purposes.

This is the ideal technique; however, it is very time consuming, costly, and usually very difficult to determine the causal variables. This technique is usually related to econometric models. The Navy uses a regression model in determining the Mean Absolute Deviation (MAD) of demand, which is further used to determine the standard deviation of demand. In this model the explanatory variable or causal variable is the average demand $\bar{D}$. This model will be discussed in more detail later in this section.

The following time series techniques are defined:

A. STRAIGHT AVERAGE — This technique does nothing more than divide the total number of demands by the number of observations as dictated by the desired time interval. For example, if average monthly demand was desired, the total number of demands would be divided by the total number of months for which demand data was available.

B. MOVING AVERAGE — In laymen's terms this method drops the oldest observation of demand and adds the latest and then computes the straight average. The computational form selects a number of base observations, say four months, eight months, or four quarters. Once the base is selected, a forecast is generated by computing the average of the base observations. Then when a new observation is available, the new forecast is

computed by adding to the old forecast the new observation minus the oldest observation divided by the number of base observations. Mathematically,

$$\text{NEW FORECAST} = \text{OLD FORECAST} + \frac{\text{NEWEST OBS} - \text{OLDEST OBS}}{\text{Number of base obs.}}$$

C. EXPONENTIAL SMOOTHING — This technique is used to assign different weights to different observations of demand. In the straight and moving average methods equal weight was assigned to each and every observation. In exponential smoothing this is not the case. The more recent demands are given heavier weights than the older demands. Mathematically the forecast is calculated as:

$$\text{NEW FORECAST} = \text{OLD FORECAST} + \alpha(\text{NEW OBS} - \text{OLD FORECAST})$$

which can be further simplified to

$$\text{NEW FORECAST} = (1-\alpha)\text{OLD FORECAST} + (\alpha)\text{NEW OBSERVATION}$$

where α = assigned weight to most recent observation and $0 \leq \alpha \leq 1$.

Now that the forecasting techniques have been defined it would be worthwhile to look at the advantages and disadvantages of each technique. An advantage of the straight average is the fact that it provides the maximum likelihood estimate of the mean demand when a normal distribution describes lead-time demand. In other words, the sample mean is an unbiased efficient estimator of the actual mean of the distribution. The disadvantage of using the straight average is its slow reaction to large increases or decreases in demand. To use this latter point as an argument against the use of this technique seems to be contradictory to the assumption of lead-time demand being normal with a constant mean. Since this technique produces the best estimator of

the parameter, it would seem that the more observations available the closer this estimate would be to the actual parameter regardless of the immediate consequences of a significant change in a particular demand observation. Another disadvantage of the straight average lies in its inability to adjust to changes in demand patterns caused by a varying mean.

The advantage of the moving average over the straight average is its ability to react more quickly to changes in demand patterns. The disadvantages associated with using this statistic are high computer costs in both space and time, which are due to the requirements to store all past observations over the base period.

Exponential smoothing provides an estimate of the mean which is asymptotically unbiased and it reacts more quickly to changes in demand patterns. It also requires very little computer space and time. These latter attributes make it the most desirable technique as far as the Navy Supply System is concerned.

Presently the Navy Uniform Inventory Control Procedures (UICP) uses exponential smoothing for forecasting demand; however, this forecast is refined by use of trend tests and a tracking signal filter. The trend test is used to verify if there is actually a change in demand pattern. The tracking signal is a statistic which is used to determine if the forecasting rule is performing well.

It is an algebraic sum of forecast errors. If the forecasting rule is operating properly the sum of these errors should be close to zero; if it is large, then the system is considered out of control and recomputation is performed.

So far, only techniques for forecasting the mean (average demand) have been discussed; however, the variance which is a measure of how demand varies about its average must also be forecasted. Estimating the variance accurately is critical since it has a pronounced affect on the reorder point and safety levels which of course drive the stock levels maintained. The stock levels further determine the major portion of the inventory investment cost. Presently the UICP model of the Navy estimates the variance by using the mean absolute deviation (MAD). The MAD is defined as the expected value of $|X-\mu|$ and the most commonly used estimate of MAD is the sample average of the absolute differences between the observed demands and their sample mean,

$$\sum_{i=1}^{N} \frac{|x_i-\bar{x}|}{N} .$$

The UICP model does not use this formulation but rather uses the formula, $\hat{MAD} = \alpha\bar{D}^{\beta}$, where $\bar{D}$ represents expected lead-time demand. The formula is based on a study done by Fleet Material Support Office (FMSO) where regression analysis was used to determine the coefficients α and β. This study [1] was done in 1962, and the values of α and β were determined to be 1.37 and .717, respectively.

The UICF model takes the MAD computed as above and then uses as the estimate of the standard deviation $\hat{S} = 1.25$ MAD. Once a MAD is determined for an item successive MADs are exponentially smoothed the same as the estimate of mean demand. The use of this technique for estimating the standard deviation is based on work done by R. G. Brown [2]. This technique has come under criticism by several investigators including particularly Dr. P. W. Zehna [3] when used with a probability density function other than the normal.

Using a 1000 item sample of demand data provided by the Air Force, which has been used in studies done by all services in the past several years, a 50 item subsample was selected randomly and estimates of the standard deviation and MAD ($MAD_1 = \frac{\Sigma|x-\bar{x}|}{N}$ form) were computed. The sample standard deviation was then compared with the value 1.25 (MAD_1) and a sign test performed to test the hypothesis that there was no difference between the two estimates of standard deviation. Since the difference between the two standard deviations was always positive, the sign test strongly rejected the hypothesis.

In view of the fact that the translation between the theoretical MAD and the standard deviation appears to be questionable it seems prudent to also investigate the use of the linear regression technique of estimating the MAD. Problems encountered with this technique would certainly

compound any problem associated with utilizing the MAD to estimate the standard deviation as is presently being done.

Great care must be taken when using a linear regression prediction scheme where the values of the explanatory variable (the predictions of mean monthly demand) used in estimating the least squares coefficient are not known. Using a regression model to predict beyond the range of values of the explanatory variable can lead to serious difficulties. From the random sample of 50 items, the regression estimate of the mean absolute deviation, $\hat{MAD} = 1.37 \bar{x}^{.717}$, was calculated for each item. These values were compared with the other estimate of the mean absolute deviation, MAD_1. It was noticed that $\hat{MAD}$ was always greater than MAD_1 when MAD_1 was less than one and $\hat{MAD}$ tended to be less than MAD_1 when the latter value exceeded one. The hypothesis that the two estimates of mean absolute deviation are the same was tested using the Wilcoxon Signed Rank Test, and it was strongly rejected at the 0.01 level of significance. Results of that test are found in Appendix A.

The above results combined with the earlier comparison of the sample standard deviation and 1.25 MAD_1 point out the possible source of problems experienced by the Navy in estimating variance. The wide disparity between the different estimates of the standard deviation can best be illustrated by presenting the three estimates for each of the 50 items. This is done in Table I.

TABLE I

Comparison of three methods for estimating standard deviation of demand.

S	1.25 MAD_1	1.25 $\widehat{MAD}$
0.26	0.16	0.26
7.56	5.71	3.98
0.37	0.17	0.26
0.80	0.78	0.98
0.19	0.08	0.16
15.48	13.25	11.69
0.72	0.54	0.66
544.56	247.55	57.33
0.13	0.04	0.10
43.53	29.95	13.89
16.30	12.08	6.28
2.98	2.43	2.14
2.55	1.52	1.39
6.48	4.54	3.15
0.94	0.60	0.69
260.33	163.49	53.61
3.46	1.77	1.50
57.17	35.46	25.09
11.92	5.54	3.60
0.82	0.37	0.46
1.16	0.65	0.69
2.23	1.70	1.53
1.16	1.02	1.24
3.90	2.31	1.79
1.14	0.77	0.81
1.17	0.73	0.79
0.81	0.45	0.53
107.11	90.10	47.13
6.48	3.70	2.62
16.40	7.53	4.71
4.37	3.16	2.32
0.91	0.66	0.79
0.52	0.32	0.42
29.90	21.41	9.96
3.79	3.57	3.00
4.05	3.35	2.64
0.26	0.16	0.26
4.65	3.60	2.73
3.83	2.72	2.43
27.55	17.48	7.99
4.71	3.93	3.50
5.81	5.54	5.50
26.12	17.08	7.87
4.47	3.67	2.96
13.32	6.22	3.83
18.88	7.03	3.91
33.72	22.08	10.05
0.85	0.42	0.50
1.31	1.06	1.14
166.55	90.25	23.77

The method of forecasting the two critical parameters, the mean lead-time demand and the standard deviation of lead-time demand, discussed in this section will also be compared in Section III in order to appraise the effects on inventory levels caused by their use.

III. SELECTION OF PROBABILITY DENSITY FUNCTIONS TO REPRESENT LEAD-TIME DEMAND

Whereas there has been a great deal of concern in the past several years regarding the ability to forecast lead-time demand and variance, little or no concern has been evidenced regarding the demand distribution itself. Obviously no one probability distribution function is going to fit the demand for all military items; however, one is interested in determining if there is any one distribution that can best represent demand as the real world sees it. An affirmative answer to this question would require a versatile distribution function since it is readily recognized that demand for military items covers the range from very low to very high, and frequently is erratic in nature. The second question to ask, given the first is answered affirmatively, would be the feasibility of using such a distribution. Three probability distributions are used in UICP in varying degrees to describe demand. These distributions are the Poisson for very low demand items (i.e. average annual demand ≤ 1), the negative binomial for items with annual demand in the range of two to ten, and normal for items with average annual demand greater than ten.

Prichard and Eagle [4] state that empirical demand distributions for items with low means are usually skewed to the right thereby making is desirable to use the Poisson

and negative binomial distributions to describe this type of item. They further state that the Poisson, negative binomial, and normal distributions seem to satisfactorily approximate demand for a majority of items and also indicate that price and consumer behavior usually are the causes for differentiating between the use of the Poisson and the negative binomial. That is, a cheap low demand item will be ordered in batches thereby causing a high variance and suggesting the use of the negative binomial. On the other hand, an expensive low demand item will be ordered one at a time thereby being better represented by the Poisson. Although not specifically stated, the reader is led to the assumption that the high demand items can be represented by the normal regardless of the price. It is by this reasoning that they contend a single mathematical distribution cannot represent all demand. They do not address, however, the possibility of a single distribution satisfactorily approximating these three distributions.

As mentioned previously, the Navy Supply System is using these distributions in varying degrees. A recent study completed by FMSO indicates that one Navy Inventory Control Point (ICP) is using the normal to represent all of its demand [5]. Additionally, the use of the Poisson has decreased in the past several years, as tables have been generated to use in its place. Basically when discussing demand distributions within the Navy it is the normal distribution that is being referred to.

Recently there has been a move underway to substitute the pseudo-normal distribution for the normal and the other distributions. The pseudo-normal has been recommended because of the computational advantages that it allows in the calculation of inventory reorder levels and reorder points. Because it performs about the same as the normal with regard to describing demand patterns, we will concentrate our attention on the normal distribution in this study.

As stated previously, representing military demand by a single probability distribution would require a very versatile distribution. It is felt that the gamma family of distributions best meets this qualification. Dr. Peter W. Zehna [6] states: "The family (gamma) is so extensive in shapes of densities available that it is a fairly safe assumption to make as a model for an experiment described by almost any non-negative random variable." Additionally Hoel, Port, and Stone [7] point out that in most cases involving a random variable X which is known to be positive the assumption that X has a gamma density will provide an approximation or at least an insight into the true but unknown situation. Finally, the versatility of the gamma is shown graphically in Figure 1 where curves associated with the Poisson, negative binomial, and normal distributions are displayed in 1A and the curves in 1B represent a particular set of gamma distributions obtained

by varying the parameters A and α. These parameters referred to are called the shape (A) and scale (α) parameters of the gamma function and will be discussed in more detail later.

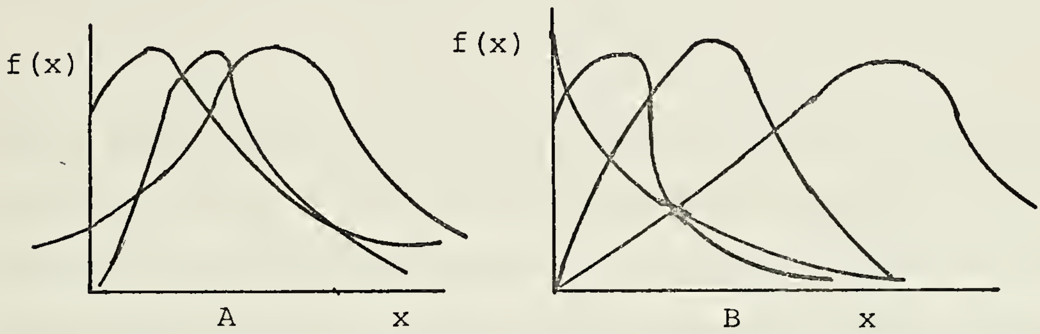


Figure 1

A true but unknown situation seems to describe more than adequately the situation inventory managers find themselves in when trying to predict how demand is going to occur. Therefore it seems appropriate that the gamma should be investigated as to its ability to represent demand of military items and to be compared with the normal as to which better describes the situation. It is precisely this objective that the remainder of this section is devoted to.

Using the sample of 50 items chosen from the 1000 item sample of Air Force data, the monthly mean ($\overline{X}$), standard deviation (S) and mean absolute deviation ($\widehat{MAD}$) were

computed based on 57 months of past demand. Additionally, the gamma parameters A and α were computed using the method of moments in the following manner:

$$\text{FOR} \quad \Gamma(x,A,\alpha) = \frac{\alpha}{\Gamma(A)} (\alpha x)^{A-1} e^{-\alpha x}; \; E(x) = A/\alpha, \; V(x) = A/\alpha^2$$

THEN solving for A and α

$$\bar{x}/S^2 = \alpha \qquad \text{and} \qquad \bar{x}^2/S^2 = A$$

These parameters were computed mechanically by the computer which at the same time was performing a goodness of fit test for two different cases. In the first case the normal distribution was fit to the data using the sample mean and the sample standard deviation which are the maximum likelihood estimators for the parameters of the normal distribution. In the second case the gamma distribution was fit to the data using the sample mean and variance to compute the estimates of the gamma parameters A and α. A Kolmogorov-Smirnov (K-S) test was performed in each case resulting in a strong rejection of the normal hypothesis and an acceptance of the majority of items under the gamma hypothesis. Table II provides a summary of the results of the K-S test.

TABLE II

Results of Kolmogorov-Smirnov test on 50 randomly selected items.

Distribution Fitted	No. Items Accepted	No. Items Rejected	Percent of Items Accepted
Normal($\bar{x}$,S^2)	0	50	0%
Gamma (A,α)	37	13	74%

A listing of results obtained in both cases is provided in Appendix B.

The necessity of fitting a probability distribution to demand data comes from the requirement to protect against running short of stock. In inventory models protecting against shortages is achieved by ordering stock before the amount on hand falls too low. The procurement policies specify that orders will be placed as soon as the stock assets reach or fall below a given level called the reorder level. The higher the reorder level, r, the lower will be the probability that the total demand during the lead-time exceeds r. Define the risk to be this probability and let its complement be called the protection level.

$$\text{RISK} = \Pr\,[\text{Total lead-time demand} > r]$$

The reorder level is adjusted up or down with the degree of adjustment depending on the desired protection level and the probability distribution of lead-time demand. The amount of material ordered, the reorder quantity, is usually constrained by the administrative cost to place an order, the cost of holding that amount of material in inventory and the cost to the system of being out of stock when a demand occurs. These costs are normally referred to as ordering, holding, and shortage costs and are not easily obtainable. Consequently they are usually a product of the particular inventory system.

The Navy uses a modification of Hadley and Whitin's lot size—reorder point model ($\langle Q,r \rangle$ model) defined as follows:

1. Let I be the annual holding cost rate per unit of stock, Ax the fixed cost of placing an order, C the cost per unit for item i, τ_i mean annual demand for item i, and π the shortage cost for each unit short. (A fixed shortage cost is not assigned for each item; rather, the penalty π is simply manipulated to meet budget and risk constraints.) If H(r) is the probability that the total lead-time demand for item i will exceed r, the reorder level is determined from the equation

$$H(r_i) = \frac{IC_iQ_i}{IC_iQ_i + \pi\tau_i}$$

2. The reorder quantity, Qi, for item i is taken to be the maximum value among (1, Qi, $\tau_i/4$). Where

$$Q_i = \sqrt{\frac{2\tau_i Ax}{IC_i}}$$

is the well known Wilson lot size.

It is important to note that under the Navy UICP model the computation of Q_i is completely independent of the reorder point and the risk, thereby making Q_i independent of the demand probability distribution.

Prior to comparing the UICP model under the assumptions of normal versus gamma distribution of lead-time demand it is necessary to obtain a feeling for the relative difference between the theoretical reorder points assigned by the gamma versus those assigned by the normal. For an illustrative purpose three items were chosen from the 50 item sample. Table III shows the results of

comparing the empirical, gamma, and normal distributions. Appendix D gives the graphic representations of the empirical cumulative distributions from which the actual protections were determined. The two normal distributions differ only in the choice of standard deviations. In one case the sample standard deviation is used and in the other case SD is computed using the regression formula for the MAD as discussed in Section II. The two sets of figures for each distribution represent 80 and 50 percent protection levels respectively. Table III indicates the gamma does a much better job at approximating the empirical distribution for the first two items than does either of the normal distributions. For the third item the gamma, and the normal using SD, both do a good job of approximating the empirical distribution at the 80 percent protection level. At the 50 percent protection level only the normal with SD does well. At this level the gamma underprotects as much as the normal using S overprotects. Looking at a histogram of this item (Figure 2) it can be seen that the gamma is not sensitive enough to the three large monthly demands (662, 757, 1733) and the normal is too sensitive. This situation seems to be the crux of the inventory problem. Should the system over-react to peaks in demand or should it ignore them? Obviously it costs inventory dollars to over-react such as in this case of stocking 71 more units than required;

TABLE III

COMPARISON OF RISKS

	$\bar{x}$=3.21 S=7.5 SD=3.98		$\bar{x}$=14.4 S=15.5 SD=11.69		$\bar{x}$=120.5 S=258.0 SD=53.61	
Distribution	Reorder Level	Actual Protection	Reorder Level	Actual Protection	Reorder Level	Actual Protection
Empirical .8	4	.807	22	.824	171	.807
($\bar{x}$, S) .5	0	.684	10	.526	50	.509
Normal .8	10	.910	28	.877	368	.930
($\bar{x}$, S) .5	4	.807	15	.631	121	.720
Normal .8	7	.895	25	.877	166	.790
($\bar{x}$,SD) .5	4	.807	15	.631	121	.720
Gamma .8	4	.807	26	.877	181	.825
(A, α) .5	1	.720	10	.526	16	.263

$\bar{x}$ = Mean

S = Sample standard deviation

SD = Computed standard deviation (SD = 1.25*MAD)

MAD = $1.37\ \bar{x}^{.717}$

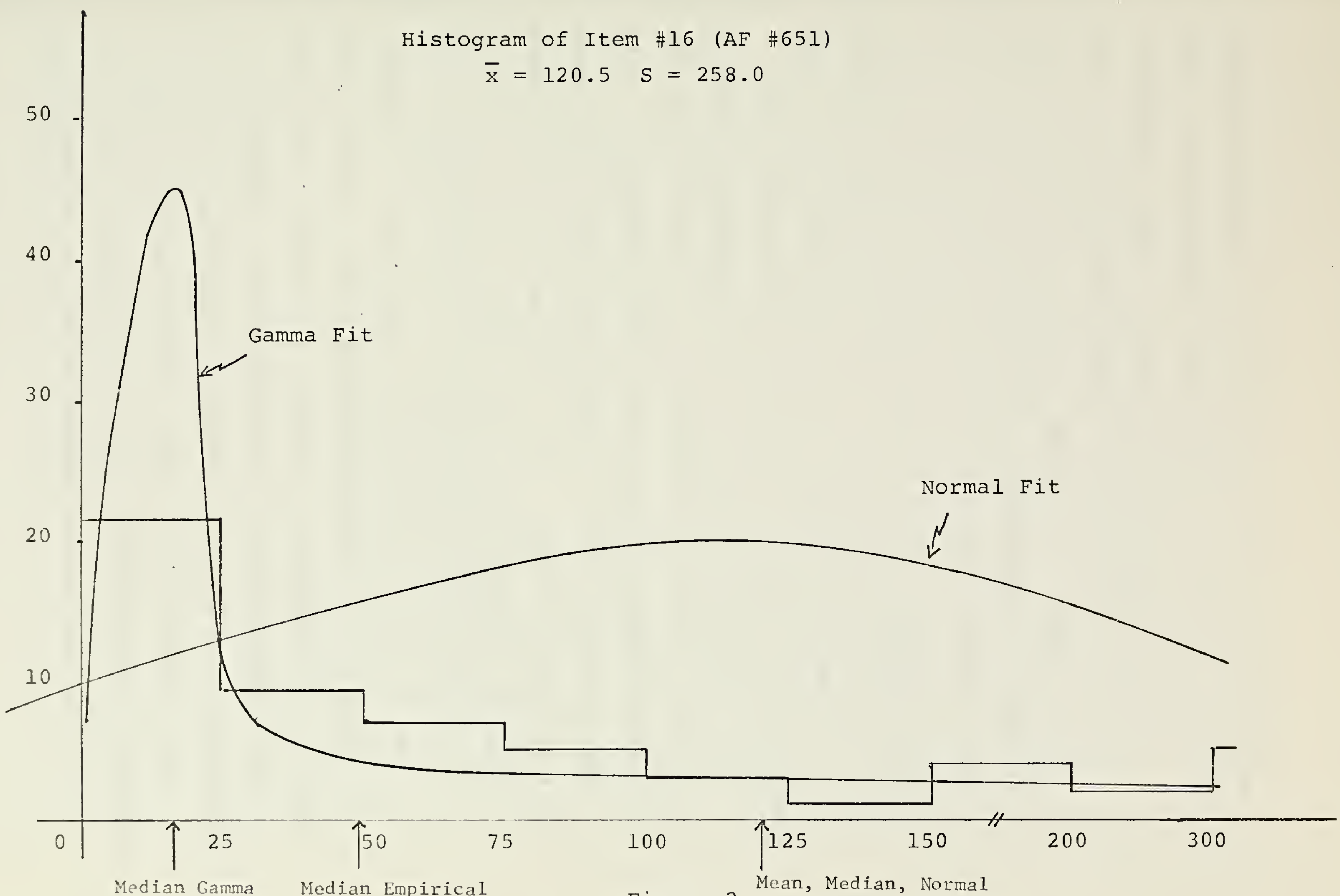

Histogram of Item #16 (AF #651)
$\bar{x}$ = 120.5   S = 258.0
50
40
30
20
10
0
25
50
75
100
125
150
200
300
Gamma Fit
Normal Fit
Median Gamma
Median Empirical
Mean, Median, Normal

Figure 2

however, how does one measure the cost of the shortages one can anticipate by only stocking 16 units. It is interesting to note the effect the computed standard deviation, SD, has on this item. The value for SD is approximately 20 percent of the value S. This is not difficult to understand when one considers the fact that in the computation of the MAD the mean is raised to the .717 power and then multiplied by 1.37. Except for mean monthly demands less than 3.04 this formulation will always give a standard deviation less than the mean.

The final comparison between the gamma and the normal is made with regards to the inventory levels and the associated costs that are generated under each distribution. A FORTRAN program was written to compute reorder levels and reorder quantities for each of the 50 items chosen from the demand data base. These values were then used to generate holding costs. In addition, shortages were accumulated and an overall unit effectiveness was computed. Overall unit effectiveness is defined as the total number of units satisfied divided by the total number of units demanded. Appendix C contains three programs to accommodate the computation of the gamma cdf, and the use of the exponentially smoothed average and the predicted standard deviation for the normal. Two different runs were made to determine the impact that different parameters would have on the levels computation and associated costs.

The procedures of the program for the first run are as follows:

1) Read the following values:
   Holding Rate = XI = .01/month
   Ordering Cost= AX = $25.

2) Generate unit price by randomly selecting values between one cent and 50 dollars. The same seed for the random number generator was used to obtain the same sequence of unit costs each time the program is run.

3) Compute the means, variances, and standard deviations using the first 21 months of demand history.

4) Compute the initial reorder levels and quantities and randomly set the initial assets on hand between r+1 and r+Q. It is in this step where the computation of r is affected by both the probability distribution and the parameters chosen and the Q by the particular forecasting technique chosen (i.e. sample mean, exponential smoothing, etc.).

5) Run the program for the remaining 36 months comparing each month's actual demand with the available stock on hand and computing units short, if any, total holding cost and finally overall unit effectiveness.

6) The final step is to compute the observed protection. This is done by dividing the total number of lead-time periods in which shorts do not occur by the total number of lead-time periods.

The first case was run three times for protection levels of 30, 50, 90 percent for each of the three distributions; the normal using the sample mean and sample standard deviation; the normal using the exponentially smoothed average and the exponentially smoothed estimate of the standard deviation; and finally the gamma using the parameters estimated by the method of moments. The results are provided in Table IV.

TABLE IV

OBSERVED PROTECTION, TOTAL HOLDING COST, TOTAL UNITS SHORT, AND OVERALL UNIT EFFECTIVENESS FOR VARIABLE r and Q = MAX (1.0, Q_w, $\tau/4$).

Desired Protection Level = 30%

	Normal ($\bar{x}$,S)	Gamma (A,α)	Normal (SMO,SD)
Observed Protection	25.81%	19.67%	39.29%
Total Holding Cost	42,379	42,217	55,734
Total Units Short	1,661	1,732	1,311
Overall Unit Effectiveness	85.73%	85.12%	88.74%

Desired Protection Level = 50%

Observed Protection	43.75%	31.15%	56.90%
Total Holding Cost	57,075	42,897	61,236
Total Units Short	763	1,474	845
Overall Unit Effectiveness	93.44%	87.33%	92.74%

Desired Protection Level = 90%

Observed Protection	59.38%	58.46%	70.00%
Total Holding Cost	107,195	67,442	74,709
Total Units Short	454	498	409
Overall Unit Effectiveness	96.10%	95.72%	96.49%

Significant among the results depicted in Table IV are the high unit effectiveness corresponding to the low observed protection levels. In fact the observed protections fell short of the desired levels in all but two cases where the normal distribution was used with the parameter estimates SMO and SD at the 30 and 50 percent desired protection levels. These results point out that the system can satisfy most of the demands (high unit effectiveness) while achieving very low protection levels. Both the short leadtime (taken to be one month in this evaluation) and the large values of the reorder quantities Q relative to the reorder levels r account for this phenomenon. For in these circumstances the system rarely relies on the reorder level to protect against stockouts. In most cases the available stock is greater than the reorder level.

Both the protection level and the unit effectiveness figures obtained by the gamma distribution fall below the figures obtained by the normal distributions. At first this seems an indictment against use of gamma distribution. However, one must consider the extra cost of achieving this higher protection. That is, one must ask if the trade off is cost effective.

To obtain higher protection levels the normal distributions must be choosing higher reorder levels on the average. These higher protection levels result in higher investment and holding costs. Thus a comparison of the

holding costs gives an indication of the cost of achieving the higher protection. Although no definite conclusions can be drawn from the values in Table IV there are indications that the gamma performs better than the normal with the same estimates of the mean and standard deviation at the higher protection levels when measured in terms of unit effectiveness per dollar of holding cost. Also, when comparing the two normal cases, the one with the exponentially smoothed estimate appears to perform better than the other using unit effectiveness per dollar of holding cost.

In order to better assess the effects of the type of probability distribution on the measures of effectiveness and the costs, the reorder quantities were all forced to be Q=1. This allows us to focus more clearly on the impact of the reorder levels determined by the two types of distribution. Also only the two cases where the sample mean and the sample standard deviation were used to estimate the parameters were considered so that any differences could be contributed to the fits of the probability distributions. Table V shows the results obtained with the reorder quantities set at one. Only the observed protection levels and the holding costs are presented in Table V because when Q=1, the protection level should be a better measure of performance than unit effectiveness.

The dominance of the gamma over the normal is better illustrated by Figure 3. There it is clearly shown that

TABLE V

OBSERVED PROTECTION AND HOLDING COSTS AT VARIOUS LEVELS OF PROTECTION.

Desired Protection Level	30%	
Demand Distribution	Normal($\bar{x}$,S)	Gamma(A,α)
Observed Protection (OP) Total Holding Cost (THC)	25.6% 1,541	23.5% 1,399
	50%	
OP THC	62.5% 11,658	33.6% 2,265
	60%	
OP THC	68.7% 19,868	42.9% 3,352
	70%	
OP THC	75.5% 29,018	56.2% 5,876
	80%	
OP THC	79.2% 40,017	69.3% 12,377
	90%	
OP THC	83.2% 55,480	81.0% 27,154

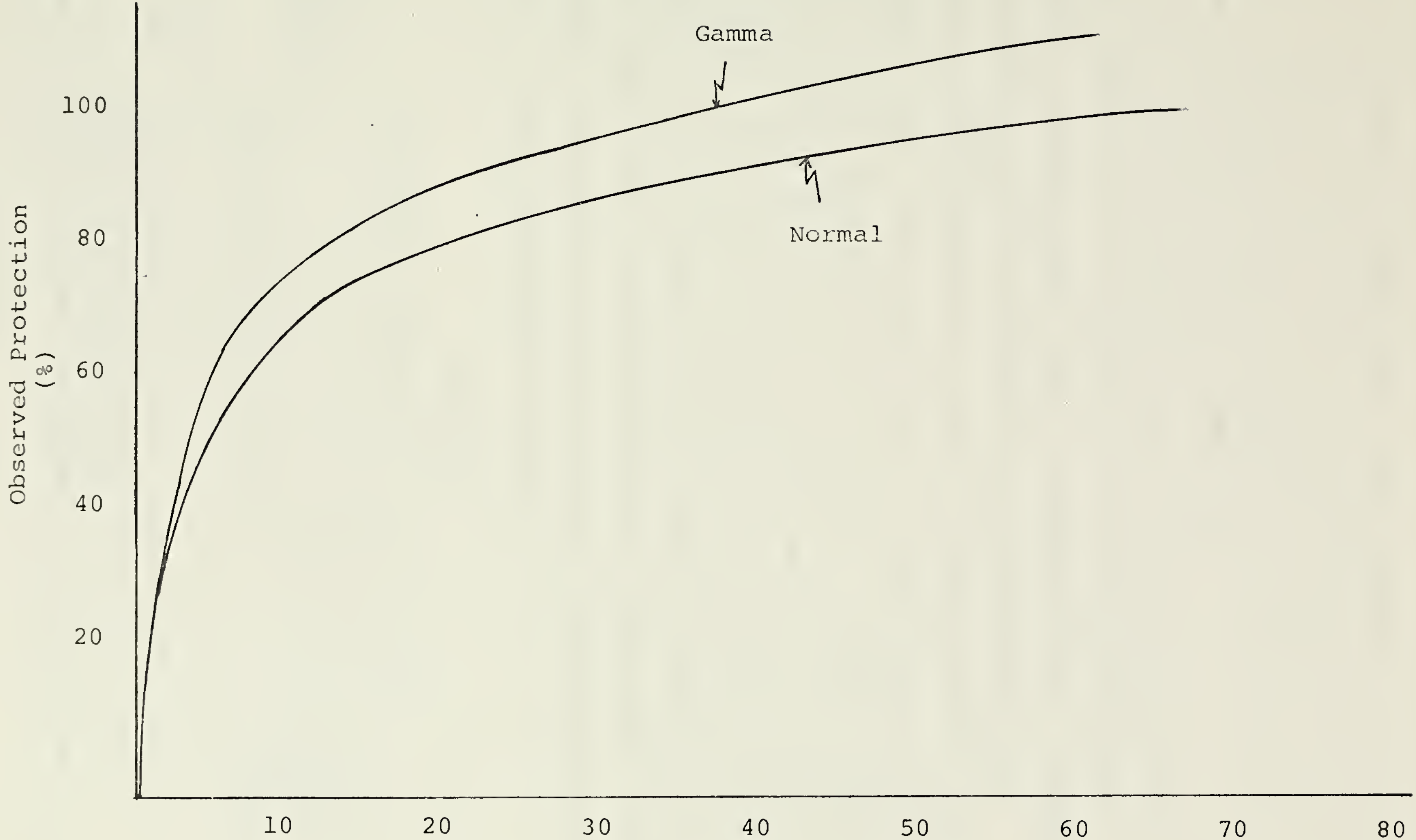


Figure 3. Holding Cost (,000)

the gamma protection is greater than the normal protection for any value of holding cost. These results coupled with the comparisons involving unit effectiveness versus dollars of holding cost support the hypothesis that the gamma is more cost effective than is the normal.

The values obtained for actual protection in Tables IV and V appear to be inconsistent since one would expect greater protection with higher inventory levels. The problem appears to be in the definition of protection and relates to the frequency at which the system reaches its reorder point. The effect that Q can have on the protection level is best shown by an example. A year's demand for one of the items of the 50 item sample is recorded in Table VI. For ease of computation the variable Q was computed as the total yearly demand divided by 4 ($Q=\tau/4$) and the reorder level was calculated from a normal distribution to give a 50 percent protection level. The initial on hand level of stock was assumed to be r+1.

TABLE VI

PERFORMANCE COMPARISON OF Q=1 vs. $Q=\tau/4$.

Demands	4	8	3	0	2	14	117	43	7	5	0	0	τ=203
$Q=\tau/4=51$													$\bar{x}$= 17
OH	14*	57	54	54	52	38	−79*	−20*	24	19	19	19	r= 17
	Protection = 1 − 2/3 = 33%												OH=18
	U.E. = 1 − 99/203 = 55%												
Q = 1.0													
OH	14*	10*	15*	18	16*	4*	−94*	−25*	11*	13*	18	18	
	Protection = 1 − 2/9 = 77%												
	U.E. = 1 − 124/203 = 39%												

where * implies an order was placed; U.E.=unit effectiveness

Finally, a discussion of the overall unit effectiveness figure used in Table IV is warranted. An important question being asked today is whether an inventory system should be geared to protect against total units short or the total number of requisitions short. Obviously, if only one unit were demanded per requisition (which is an assumption of the Q, r model) these two measures would be the same. However, when examining demand patterns for the items used in this study it was noticed that there would be many months of demand within a particular range and then two or three months of very high demand. For example, demand in the range of 50 to 100 would be received for several months and then a demand for 3300 would be received. It would be difficult to accept the assumption that the 3300 units demanded represented 3300 requisitions. The importance of this point is that if the 3300 units demanded were not on separate requisitions the value of unit effectiveness becomes distorted.

IV. METHOD OF RECORDING DEMAND

Up to this point the problems associated with forecasting demand have been defined as the ability to describe the probability density function of demand and the selection of the proper forecasting scheme to predict the parameters of that density function. One of the major factors contributing to the complexity of these problems is the nature of demand patterns. An examination of the 1000 item sample used in this study indicated that in the majority of cases the demand patterns were very erratic. The primary effect of erratic demand is to cause large variances thereby rendering the task of fitting a probability distribution to the demand data, particularly difficult. More important to the inventory manager, however, is the effect these large variances have on his inventory levels.

One of the major factors causing demand patterns as the inventory manager sees them is the demand reporting system that is utilized. If a ship requires a repair part, then the demand is created at the shipboard level at a particular point in time. Under current procedures there are four echelons of supply from which the ship can obtain the repair part required. Therefore the initial point at which the demand is recorded depends on where the requirement is satisfied. The four echelons referred to are the ship itself, a mobile logistics support ship, a stock point and

finally, an inventory control point. These echelons are critical in determining when a demand gets recorded at the ICP, which is the inventory manager and has the responsibility of setting system inventory levels. For example, if a shipboard demand occurs on the first of the month and the ship can satisfy the demand from its own stocks then the demand is recorded at the shipboard level only at that point in time.

If the demand did not reduce the onboard stocks to the ship's reorder point this particular demand would not be recorded at the stock point until sufficient demands occurred to reduce the stock to the reorder point, at which time the ship would place a replenishment demand on the stock point. Another factor affecting the time at which the demand is reorded at the stock point is whether the ship is at sea or in port since the ship has no means of communicating (except for high priority requirements) directly with the stock point if it is at sea. The point here is that the demand that a stock point observes could represent demand for that day or an accumulation of demand over a period of time. Since stock points report daily to the ICP's this is also the situation that the ICP inventory manager faces. There is such a large population of ships that enter and leave port at different times it would seem that over a period of time, the system demand would smooth itself. However, a review of actual data patterns does not substantiate this. Thus, an investigation

into the method of collecting data seems warranted. Since time would not permit a thorough investigation it was decided to look at the effects of this situation through a simple simulation model in which demands for a single item were generated from seven different ships. It was assumed that the time between demands is exponentially distributed. The time at sea was assumed to be uniformly distributed between 0 and 45 days. Based on these assumptions the days on which demands occurred were randomly generated for each ship. Also the dates of arrival and departure from port were randomly generated, and initially all ships were considered at sea. It was also assumed that all demands incurred while the ship was at sea are reported to the stock point upon arrival in port. Additionally, any demands occurring while the ship was in port were reported to the stock point on the day they occurred.

The alternative method of recording demand investigated in this simulation had the ship report the demand directly to the ICP on the day it occurred. Inherent in this alternative is the assumption that the ship has the capability of reporting demand directly to the ICP which is recognized as unrealistic at this time. However, since the ships currently keep records of daily demand through the 3M (Material, Management and Maintenance) system, it was felt that the above alternative was possible. Thus, we are comparing consumption data vice demand data as currently reported in the Navy.

The simulation was run for a two year period with the demand recorded by month at both the ICP and stock point. Additionally, the mean and variance of demand over this two year period were computed for each reporting system. The results of this simulation are presented in Table VII.

TABLE VII

SIMULATED MONTHLY DEMAND
AS RECORDED AT ICP AND NSC

DATA	YEAR	MONTHLY DEMAND											
Consumption Data	1	9	18	18	15	17	17	18	19	15	15	23	22
	2	19	11	16	12	17	18	4	12	20	12	15	8
Demand Data	1	4	7	30	4	11	16	23	23	20	17	20	24
	2	7	17	9	13	8	21	2	4	40	13	22	8

Mean of Consumption Data = 15   Mean of Demand Data = 15

Variance of Consumption Data = 32   Variance of Demand Data = 84

As expected, the alternative of reporting demand directly to the ICP (consumption data) shows a smoother pattern than that reported to the stock point (demand data). The critical result however is that the variance of demand at the stock point was approximately $2\frac{1}{2}$ times greater than the variance of demand recorded at the ICP. With smoother or more regular demand patterns and smaller variances the task of forecasting future demands should be easier. It would

be presumptuous to draw any conclusions based on such tentative results. However, further investigation into the use of consumption data does seem appropriate.

V. CONCLUSIONS

A. DEMAND FORECASTING TECHNIQUES

The following recomendations are made with regard to the forecasting schemes to be utilized to predict the mean and variance of the lead-time demand.

1) That the sample variance (S^2) be used for predicting variance of lead-time demand regardless of the probability distribution chosen. There appears no statistical justification for using the Mean Absolute Deviation regardless of the particular estimate of MAD used. Since there was an apparent strong relationship between the sample MAD and the mean quarterly demand when the study was conducted at FMSO in 1962 it might be worthwhile to perform such a study with current demand. The use of the coefficients of 1.37 and .717 computed in 1962 certainly does not appear warranted.

2) The results of this study show no distinct preference for the use of a sample average or the exponentially smoothed estimate of the mean.

B. PROBABILITY DENSITY FUNCTION

It is recommended that the current procedures of using three probability density functions to represent lead-time demand be replaced by the procedure of using the single distribution of gamma. This recommendation is based on the following conclusions:

1) The gamma exhibits the versatility to represent low demand items as well as high demand items. Additionally the gamma is a non-negative probability density function which ensures that all of the probability mass will be represented by actual demand (i.e. no such thing as a negative demand such as is the case with the normal).

2) The gamma appears to be less sensitive to peaks in demand and therefore consistently assigns lower reorder levels creating much lower inventory holding costs. The argument here is that unless the demand pattern is smooth, stocking material under the normal assumption tends to put higher reorder levels than required to meet most demands. These higher reorder levels naturally result in higher protection levels but at the same time they require the system to carry a lot of stock and therefore incur high holding and investment costs to protect against infrequent demands.

3) Finally, the conversion from the present system to the gamma would require little effort since the incomplete gamma function which gives the complementary cumulative distribution is already programmed and tabulated. Additionally using the method of moments to compute the gamma parameters requires minimum machine effort.

APPENDIX A

VALUES OF S, SA, MAD_1 AND $\hat{MAD}$ USED IN SIGN, AND SIGNED RANK TEST

S	SA	MAD_1	$\hat{MAD}$
0.26	0.16	0.13	0.21
7.56	5.71	4.57	3.19
0.37	0.17	0.14	0.21
0.80	0.78	0.62	0.79
0.19	0.08	0.07	0.13
15.48	13.25	10.60	9.35
0.72	0.54	0.43	0.53
544.56	247.55	198.04	45.86
0.13	0.04	0.03	0.08
43.53	29.95	23.96	11.11
16.30	12.08	9.67	5.02
2.98	2.43	1.95	1.71
2.55	1.52	1.22	1.11
6.48	4.54	3.63	2.52
0.94	0.60	0.48	0.56
260.33	163.49	130.79	42.89
3.46	1.77	1.42	1.20
57.17	35.46	28.37	20.07
11.92	5.54	4.43	2.88
0.82	0.37	0.29	0.37
1.16	0.65	0.52	0.56
2.23	1.70	1.36	1.22
1.16	1.02	0.82	0.99
3.90	2.31	1.85	1.43
1.14	0.77	0.62	0.65
1.17	0.73	0.58	0.63
0.81	0.45	0.36	0.42
107.11	90.10	72.08	37.70
6.48	3.70	2.96	2.10
16.40	7.53	6.02	3.77
4.37	3.16	2.52	1.86
0.91	0.66	0.53	0.63
0.52	0.32	0.26	0.34
29.90	21.41	17.13	7.97
3.79	3.57	2.85	2.40
4.05	3.35	2.68	2.11
0.26	0.16	0.13	0.21
4.65	3.60	2.88	2.18
3.83	2.72	2.18	1.95
27.55	17.48	13.99	6.39
4.71	3.93	3.15	2.80
5.81	5.54	4.43	4.40
26.12	17.08	13.66	6.30
4.47	3.67	2.94	2.37
13.32	6.22	4.98	3.06
18.88	7.03	5.63	3.13
33.72	22.08	17.66	8.04
0.85	0.42	0.33	0.40
1.31	1.06	0.85	0.91
166.55	90.25	72.20	19.02

Results: Sign Test – Ho: S = SA. Since (S–SA) > 0 in all cases strongly reject hypothesis.
Wilcoxon Signed Rank Test – Ho: $MAD_1 = \hat{MAD}$
Test statistic is T = –171.5 for negative sign at α = .01
T_c = 612.5 from Ref. [8]. Therefore strongly reject hypothesis.

APPENDIX B

KOLMOGOROV–SMIRNOV TEST

NORMAL		GAMMA	
PROB = 0%	PROB = 0%	PROB = 99.6%	PROB = 100%
0	0	100	100
0	3.2	100	100
0	0	6.6*	62.6
0	0	100	100
5.8	0	44.9*	1.9*
0	0	65.2	99
0	0	0*	45.8*
0	0	100	100
0	0	96.2	99.7
0	0	66.6	88
0	0	99.5	99.9
0	0	100	99.6
0	0	100	55*
0	0	98	69.8
0	0	1.8*	92.6
0	.8	100	72.7
0	0	.5*	42.2*
0	0	18.1*	100
0	0	100	16.7*
0	0	100	100
0	0	100	95.5
0	0	122*	100
0	0	100	72.9
0	0	99.7	17.3*

PROB = Probability of being incorrect if hypothesis is rejected

* indicates gamma hypothesis is rejected

APPENDIX C

FORTRAN PROGRAMS ASSOCIATED WITH COMPUTATION OF INVENTORY COSTS AND UNIT EFFECTIVENESS

FORTRAN programs used in calculating values for Tables IV and V.

The three programs in this Appendix represent computations using a variable Q. For the values on Table V the programs were modified to set Q = 1.0. Additionally these basic programs were run with a counter to compute the number of periods when shortages occur and the total number of periods when orders were placed. This later modification was necessary to calculate the observed protection levels.

Important labels used in the programs and not previously defined are as follows:

BAR	Mean monthly demand
VAR	Variance of monthly demand
PT	Desired protection level
TSHT	Total units short
THCT	Total holding cost
OUE	Overall unit effectiveness

FORTRAN PROGRAM FOR GAMMA (A,α)

```
      REAL*8 A,XX
      DIMENSION IDEM(57),DEM(57),Z(300)
      AX=25.
      XSUM=0.
      XSHT=0.
      THCT=0.
      XI=.01
   20 READ(5,10,END=50) (IDEM(I),I=1,57)
   10 FORMAT(12I5)
      DO 500 I=1,57
  500 DEM(I)=IDEM(I)
      PT=.5
      Z(1)=0.
      N=21
      M=N
      K=21
      HCT=0.
      TSHT=0.
      TSUM=0.
      DEV=0.
      IX=1237561
      CALL RANDU(IX,IX,YFL)
      COST=YFL*50.
      HC=(XI*COST)
    2 SUM=0.
      SSQ=0.
      DO 3 I=1,N
      SUM=SUM+DEM(I)
    3 SSQ=SSQ+DEM(I)**2
      BAR=SUM/FLOAT(N)
      IF(BAR.LE.0.0)GO TO 16
      VAR=(SSQ-BAR*SUM)/FLOAT(N-1)
      A=BAR**2/VAR
      ALPHA=BAR/VAR
      DO 4 I=2,300
      Z(I)=Z(I-1)+1.
      XX=Z(I-1)*ALPHA
      CALL GAMA(A,XX,GAM,B,ER)
      Y=1.-GAM
      IF(Y-PT)4,5,5
    4 CONTINUE
    5 RN=Z(I-1)
      IF(RN.LT.1.0)RN=1.0
      ALAMD=BAR*12.
      XNUM=2*(ALAMD*AX)
      DNOM=XI*COST
      QW=SQRT(XNUM/DNOM)
      XLAM=ALAMD*3.
      IF(QW.GT.XLAM)QW=XLAM
      DBAR=ALAMD/4.
      Q=AMAX1(1.0,QW,DBAR)
      IF(K.LT.N)GO TO 7
      IAN=RN+YFL*(Q+1.)
    7 M=N+1
      IF(IAN-DEM(M))8,9,9
    8 SHRT=(DEM(M)-IAN)
      TSHT=TSHT+SHRT
      ACH=FLOAT(IAN)/2.
      HCT=HCT+ACH*HC
      IBUY=RN/Q
      BUY=FLOAT(IBUY+1)*Q
      IAN=BUY
      GO TO 16
    9 OH=IAN-DEM(M)
      AOH=(IAN+OH)/2.
      HCT=HCT+AOH*HC
      IF(OH.GE.RN)GO TO 15
      IBUY=(RN-OH)/Q
      BUY=FLOAT(IBUY+1)*Q
      IAN=BUY+OH
      GO TO 16
```

```
 15 IAN=0H
 16 N=N+1
    TSUM=TSUM+DEM(M)
    IF(N-57)2,17,17
 17 THCT=THCT+HCT
    XSUM=XSUM+TSUM
    XSHT=XSHT+TSHT
    GO TO 20
 50 CUE=1.-XSHT/XSUM
    WRITE(6,200)XSHT,THCT,CUE
200 FORMAT(10X,'XSHT=',F9.2/10X,'THCT=',F10.2/10X,
   *10X,'OUE=',F6.4)
    STOP
    END
```

FORTRAN PROGRAM FOR NORMAL (SMO, SD)

```
      DIMENSION IDEM(57),DEM(57),Z(300)
      AX=25.
      XSUM=0.
      XSHT=0.
      THCT=0.
      XI=.01
   20 READ(5,10,END=50) (IDEM(I),I=1,57)
   10 FORMAT(12I5)
      DO 500 I=1,57
  500 DEM(I)=IDEM(I)
      PT=.5
      Z(1)=0.
      N=21
      M=N
      K=21
      HCT=0.
      TSHT=0.
      TSUM=0.
      DEV=0.
      IX=1237561
      CALL RANDU(IX,IX,YFL)
      COST=YFL*50.
      HC=(XI*COST)
    2 SUM=0.
      SSQ=0.
      DO 3 I=1,N
      SUM=SUM+DEM(I)
    3 SSQ=SSQ+DEM(I)**2
      BAR=SUM/FLOAT(N)
      IF(BAR.LE.0.0)GO TO 16
      VAR=(SSQ-BAR*SUM)/FLOAT(N-1)
      S=SQRT(VAR)
      RN=W*S+BAR
      IF(RN.LT.1.0)RN=1.0
      ALAMD=BAR*12.
      XNUM=2*(ALAMD*AX)
      DNOM=XI*COST
      QW=SQRT(XNUM/DNOM)
      XLAM=ALAMD*3.
      IF(QW.GT.XLAM)QW=XLAM
      DBAR=ALAMD/4.
      Q=AMAX1(1.0,QW,DBAR)
      IF(K.LT.N)GO TO 7
      IAN=RN+YFL*(Q+1.)
    7 M=N+1
      IF(IAN-DEM(M))8,9,9
    8 SHRT=(DEM(M)-IAN)
      TSHT=TSHT+SHRT
      AOH=FLOAT(IAN)/2.
      HCT=HCT+AOH*HC
      IBUY=RN/Q
      BUY=FLOAT(IBUY+1)*Q
      IAN=BUY
      GO TO 16
    9 OH=IAN-DEM(M)
      AOH=(IAN+OH)/2.
      HCT=HCT+AOH*HC
      IF(OH.GE.RN)GO TO 15
      IBUY=(RN-OH)/Q
      BUY=FLOAT(IBUY+1)*Q
      IAN=BUY+OH
      GO TO 16
   15 IAN=OH
   16 N=N+1
      TSUM=TSUM+DEM(M)
      IF(N-57)2,17,17
   17 THCT=THCT+HCT
      XSUM=XSUM+TSUM
      XSHT=XSHT+TSHT
      GO TO 20
   50 OLE=1.-XSHT/XSUM
```

```
      WRITE(6,200)XSHT,THCT,QUE
200   FORMAT(10X,'XSHT=',F9.2/10X,'THCT=',F10.2/10X,
     *10X,'QUE=',F6.4)
      STOP
      END
```

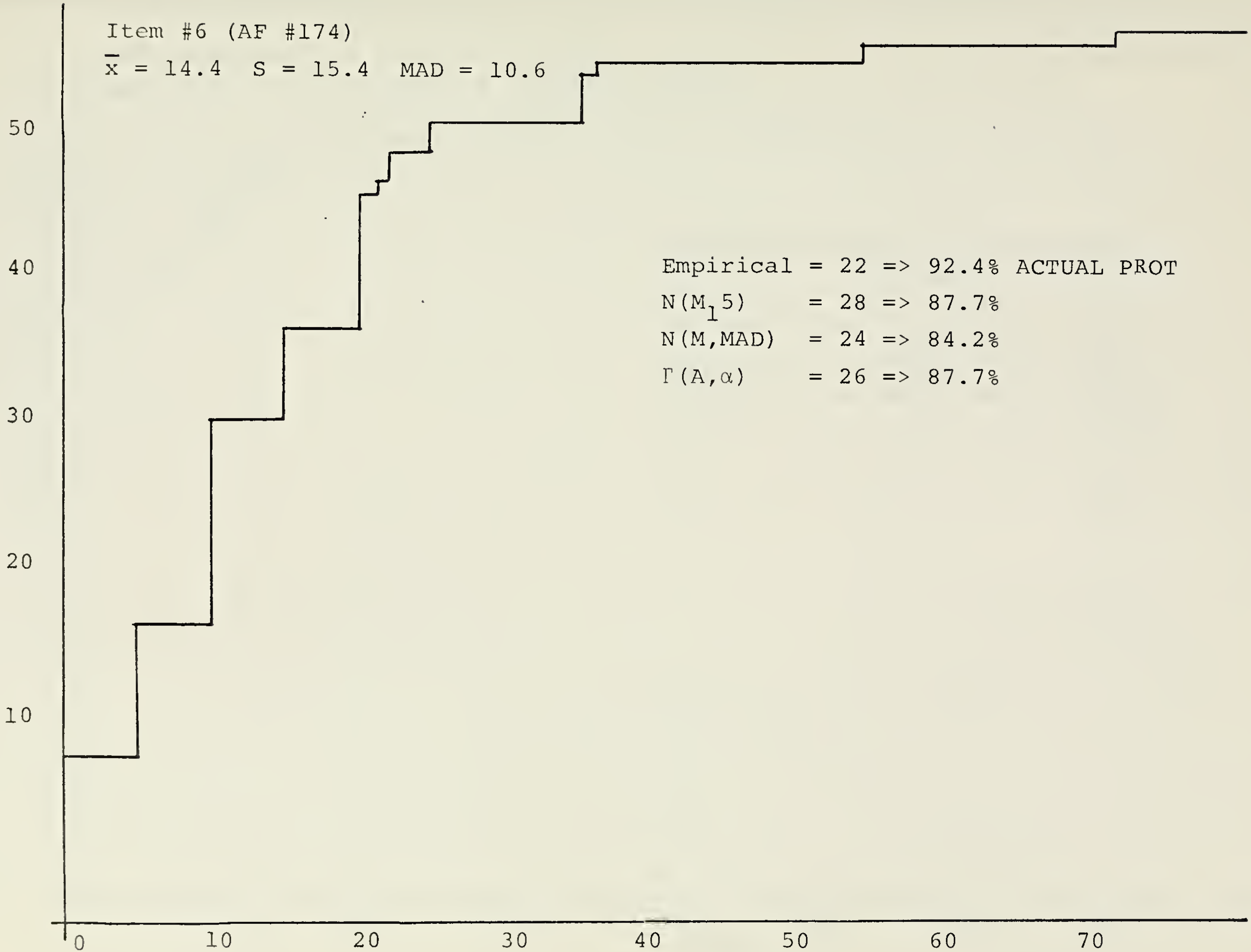
FORTRAN PROGRAM FOR NORMAL ($\overline{x}$,S) DISTRIBUTION

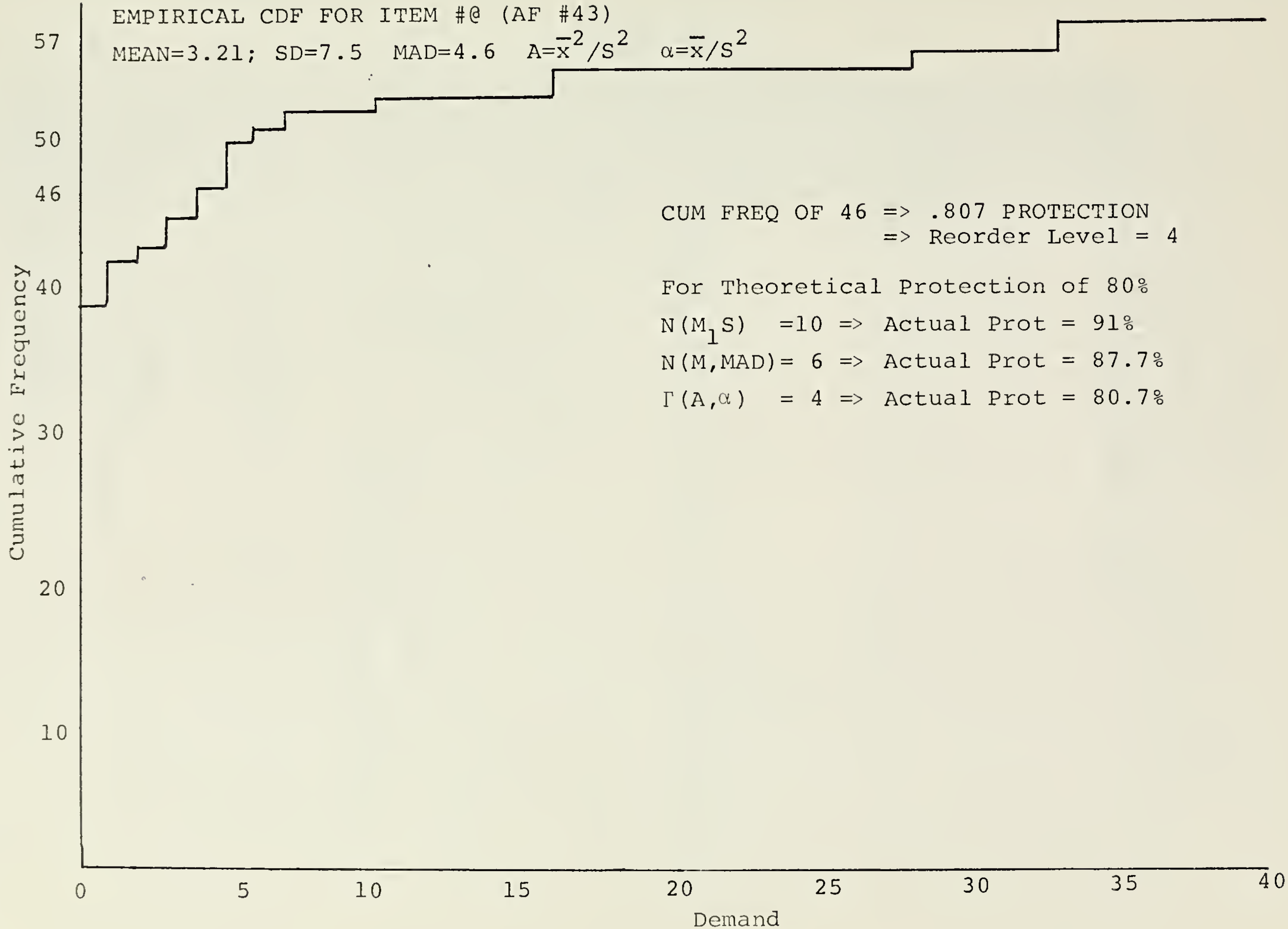
```
      DIMENSION IDEM(57),DEM(57),Z(300)
      A=0.323
      B=0.717
      AX=25.
      XSUM=0.
      XSHT=0.
      THCT=0.
      XI=.01
   20 READ(5,10,END=50) (IDEM(I),I=1,57)
   10 FORMAT(12I5)
      DO 500 I=1,57
  500 DEM(I)=IDEM(I)
      PT=.5
      Z(1)=0.
      N=21
      M=N
      K=21
      HCT=0.
      TSHT=0.
      TSUM=0.
      DEV=0.
      IX=1237561
      CALL RANDU(IX,IX,YFL)
      COST=YFL*50.
      HC=(XI*COST)
    2 SUM=0.
      SSQ=0.
      DO 3 I=1,N
      SUM=SUM+DEM(I)
    3 SSQ=SSQ+DEM(I)**2
      BAR=SUM/FLOAT(N)
      IF(BAR.LE.0.0)GO TO 16
      VAR=(SSQ-BAR*SUM)/FLOAT(N-1)
      XY=A+B*(ALOG(BAR))
      AMAD=EXP(XY)
      S=1.25*AMAD
      SMO=BAR
      GO TO 13
   12 DEV=ABS(DEM(M)-SMO)
      AMAD=0.2*DEV+0.8*AMAD
      S=1.25*AMAD
      SMO=0.2*DEM(M)+0.8*SMO
   13 RN=W*S+SMO
      IF(RN.LT.1.0)RN=1.0
      ALAMD=SMO*12.
      XNUM=2*(ALAMD*AX)
      DNOM=XI*COST
      QW=SQRT(XNUM/DNOM)
      XLAM=ALAMD*3.
      IF(QW.GT.XLAM)QW=XLAM
      DBAR=ALAMD/4.
      Q=AMAX1(1.0,QW,DBAR)
      IF(K.LT.N)GO TO 7
      IAN=RN+YFL*(Q+1.)
    7 M=N+1
      IF(IAN-DEM(M))8,9,9
    8 SHRT=(DEM(M)-IAN)
      TSHT=TSHT+SHRT
      AOH=FLOAT(IAN)/2.
      HCT=HCT+AOH*HC
      IBUY=RN/Q
      BUY=FLOAT(IBUY+1)*Q
      IAN=BUY
      GO TO 16
    9 OH=IAN-DEM(M)
      AOH=(IAN+OH)/2.
      HCT=HCT+AOH*HC
      IF(OH.GE.RN)GO TO 15
      IBUY=(RN-OH)/Q
      BUY=FLOAT(IBUY+1)*Q
      IAN=BUY+OH
```

```
      GC TO 16
   15 IAN=0H
   16 N=N+1
      TSUM=TSUM+DEM(M)
      IF(N-57)12,17,17
   17 THCT=THCT+HCT
      XSUM=XSUM+TSUM
      XSHT=XSHT+TSHT
      GC TO 20
   5C OUE=1.-XSHT/XSUM
      WRITE(6,200)XSHT,THCT,OUE
  200 FCRMAT(10X,'XSHT=',F9.2/10X,'THCT=',F10.2/10X,
     *10X,'OUE=',F6.4)
      STOP
      END
```

APPENDIX D

EMPIRICAL CUMULATIVE DISTRIBUTION FUNCTIONS


Item #6 (AF #174)
$\bar{x}$ = 14.4 S = 15.4 MAD = 10.6
Empirical = 22 => 92.4% ACTUAL PROT
$N(M_1 5)$ = 28 => 87.7%
N(M,MAD) = 24 => 84.2%
$\Gamma(A,\alpha)$ = 26 => 87.7%
50
40
30
20
10
0
10
20
30
40
50
60
70



EMPIRICAL CDF FOR ITEM #@ (AF #43)
MEAN=3.21; SD=7.5 MAD=4.6 $A=\bar{x}^2/S^2$ $\alpha=\bar{x}/S^2$
CUM FREQ OF 46 => .807 PROTECTION
=> Reorder Level = 4
For Theoretical Protection of 80%
$N(M_1S)$ =10 => Actual Prot = 91%
N(M,MAD)= 6 => Actual Prot = 87.7%
$\Gamma(A,\alpha)$ = 4 => Actual Prot = 80.7%
57
50
46
40
30
20
10
Cumulative Frequency
0
5
10
15
20
25
30
35
40
Demand

APPENDIX E

FORTRAN PROGRAM AND RESULTS OF SIMULATION MODEL

```
      DIMENSION ISHIP(7),A(7),SEA(7),NSEA(7),NPCRT(7),
     1NICP(7),NSC(7),NCUM(7),INC(7),INK(7)
      IDAY=0
      IX=773
      NDAY=720
      N=7
      NICPM=0
      NSCM=0
      NXEARI=0
      NXSQRI=0
      NXEARN=0
      NXSQRN=0
      NP=NDAY/30
      DO 10 I=1,7
      ISHIP(I)=0
      NSEA(I)=1
      NPCRT(I)=0
      INC(I)=0
      INK(I)=0
      NICP(I)=0
      NSC(I)=0
      NCUM(I)=0
   10 CONTINUE
      READ(5,1000)A(1),A(2),A(3),A(4),A(5),A(6),A(7)
 1000 FORMAT(7(F3.0))
      READ(5,1500)SEA(1),SEA(2),SEA(3),SEA(4),SEA(5),SEA(6),
     1SEA(7)
 1500 FORMAT(7(F5.0))
  250 CONTINUE
      DO 15 J=1,30
      IDAY=IDAY+1
      IF(IDAY.GT.NDAY)GO TO 500
      DO 20 I=1,N
      IF(IDAY.LT.NSEA(I))GO TO 18
      CALL RANDU(IX,IY,YFL)
      IX=IY
      INC(I)=-SEA(I)*ALOG(YFL)
      NPCRT(I)=IDAY+INC(I)
      NSEA(I)=NPCRT(I)+10
   18 IF(ISHIP(I).GT.IDAY.OR.ISHIP(I).LE.0)GO TO 100
      NICP(I)=NICP(I)+1
      NCUM(I)=NCUM(I)+1
      IF(ISHIP(I).GE.NPCRT(I).AND.ISHIP(I).LE.NSEA(I)),
     *GO TO 50
      CALL RANDU(IX,IY,YFL)
      IX=IY
      INK(I)=-A(I)*ALOG(YFL)
      ISHIP(I)=ISHIP(I)+INK(I)
      GO TO 20
   50 NSC(I)=NSC(I)+NCUM(I)
      NCUM(I)=0
      CALL RANDU(IX,IY,YFL)
      IX=IY
      INK(I)=-A(I)*ALOG(YFL)
      ISHIP(I)=ISHIP(I)+INK(I)
      GO TO 20
  100 IF(ISHIP(I).GT.IDAY)GO TO 20
      CALL RANDU(IX,IY,YFL)
      IX=IY
      INK(I)=-A(I)*ALOG(YFL)
      ISHIP(I)=ISHIP(I)+INK(I)
   20 CONTINUE
```

```
   15 CONTINUE
      NICPM=NICP(1)+NICP(2)+NICP(3)+NICP(4)+NICP(5)+NICP(6),
     *+NICP(7)
      NSCM=NSC(1)+NSC(2)+NSC(3)+NSC(4)+NSC(5)+NSC(6)+NSC(7)
      WRITE(6,2100)NICPM,NSCM
 2100 FORMAT('0','ICP DEMAND',I5,10X,'NSC DEMAND',I5)
      NXBARI=NXBARI+NICPM
      NXSQRI=NXSQRI+NICPM*NICPM
      NXBARN=NXBARN+NSCM
      NXSQRN=NXSQRN+NSCM*NSCM
      DO 30 I=1,7
      NICP(I)=0
      NSC(I)=0
   30 CONTINUE
      NICPM=0
      NSCM=0
      GO TO 250
  500 NXBARI=NXBARI/NP
      NXBARN=NXBARN/NP
      NXSQRI=NXSQRI/NP
      NXSQRN=NXSQRN/NP
      NVARI=NXSQRI-NXBARI*NXBARI
      NVARN=NXSQRN-NXBARN*NXBARN
      WRITE(6,2200)NXBARI,NVARI,NXBARN,NVARN
 2200 FORMAT('0','ICP MEAN',I5,5X,'ICP VARIANCE',I5,
     15X,'NSC MEAN',I5,5X,'NSC VARIANCE',I5)
      STOP
      END
```

Variable names used in the program are defined as follows:

ISHIP(I)	Day of demand for Ship I
NPORT(I)	Day of arrival in port for Ship I
NSEA(I)	Day of departure from port for Ship I
NICPM	Total monthly demand recorded at Inventory Control Point (ICP)
NSCM	Total monthly demand recorded at Naval Supply Center (NSC)
NXBARI	Mean demand recorded at ICP
NVARI	Variance of demand recorded at ICP
NXBARN	Mean demand recorded at NSC
NVARN	Variance of demand recorded at NSC

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20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

An analysis is made of current Navy procedures for assigning a probability density function to demand and the technique(s) used to forecast the parameter(s) of the particular density function chosen. The gamma density function is investigated as a possible replacement for the three density functions currently being used. A comparison of the gamma and normal density functions

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is made with regards to inventory costs, observed protection levels, and unit effectiveness. Additionally, a comparison is made, through a simple simulation model, of demand patterns generated by demand reporting as it is done today and consumption data reporting as it might be done in the future. The intent of the simulation is to provide some insight into the impact the demand reporting method might have on the variance of demand.